

## THE LAW OF COSINES

### ALGEBRA 2 WITH TRIGONOMETRY

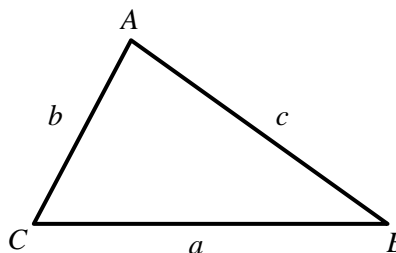
Just as there is a law that relates the sine of an angle to its opposite side, there is also one that relates the cosine to its opposite side. Its derivation is left to your precalculus course. Here we will state it without proof and use it to find missing sides and angles of a triangle.

#### THE LAW OF COSINES

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

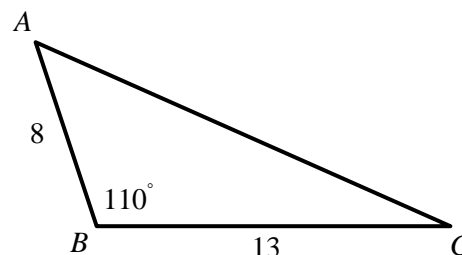
$$a^2 = b^2 + c^2 - 2bc \cos A$$



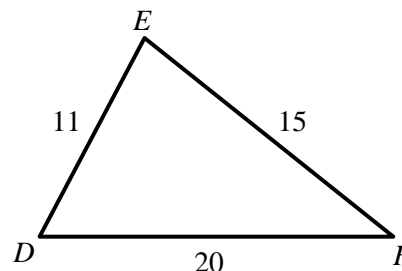
**Exercise #1:** If  $m\angle C = 90^\circ$  then the first form of the Law of Cosines results in what famous theorem? Justify your answer by substituting  $90^\circ$  for  $C$ .

The Law of Cosines, like the Law of Sines, serves two primary purposes, namely finding missing sides and angles in a triangle. These two skills will be illustrated in *Exercises #2* and *#3*, respectively.

**Exercise #2:** In  $\triangle ABC$  below,  $AB = 8$ ,  $BC = 13$ , and  $m\angle B = 110^\circ$ . Find the length of  $\overline{AC}$  to the nearest *tenth*.



**Exercise #3:** In triangle  $DEF$  below,  $DE = 11$ ,  $EF = 15$ , and  $DF = 20$ . Determine  $m\angle D$  to the nearest degree.



We can now solve a variety of scenarios, specifically when S.A.S. or S.S.S. information is given about a triangle, that we couldn't with the Law of Sines (which only applies in S.S.A. or A.A.S. situations).

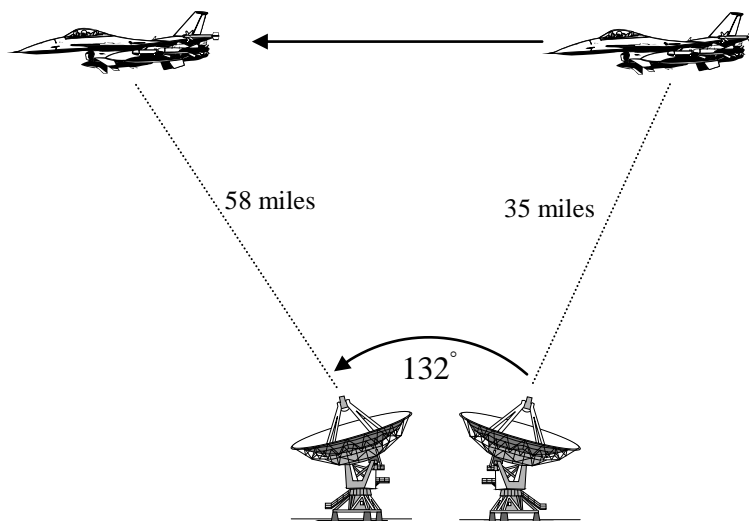
**Exercise #4:** A triangle has three sides that measure 8, 14, and 20 centimeters.

- (a) Find the measure of the largest angle of this triangle to the nearest *tenth* of a degree.

Convenient Space Below for Diagram

- (b) Using your answer from (a), determine the area of this triangle to the nearest square centimeter.

**Exercise #5:** A satellite dish can track the speed of a plane by recording the distance to the plane at two points in time and the angle through which the dish rotates. In the picture below, a satellite measured the distance to a jet at 35 miles. After 0.25 hours (15 minutes), it measured the distance to the jet at 58 miles. If the satellite rotated through an angle that measured  $132^\circ$ , determine the average speed of the plane to the nearest mile per hour.

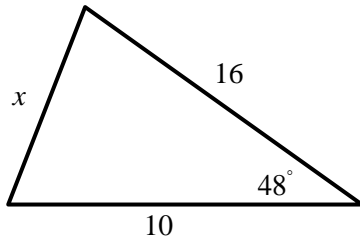


**THE LAW OF COSINES**  
**ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK**

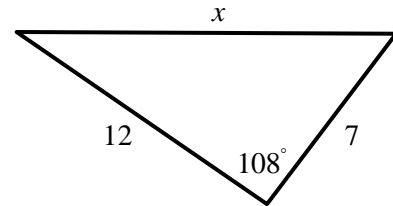
**SKILLS**

1. In each of the triangles below, a S.A.S. scenario is given. Find the value of  $x$  in each case to the nearest *tenth* using the Law of Cosines.

(a)

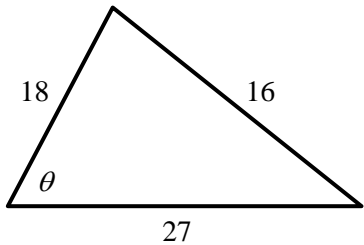


(b)

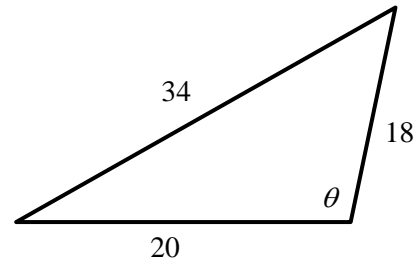


2. In each of the triangles below, a S.S.S. scenario is given. Using the Law of Cosines, find the measure of  $\theta$  as marked in each triangle accurate to the nearest *degree*.

(a)



(b)



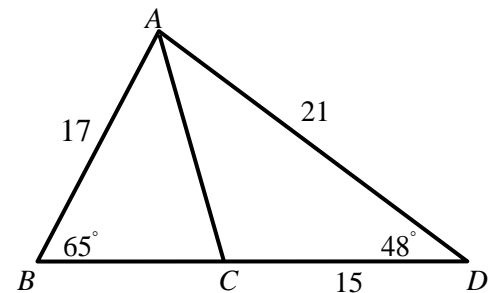
3. In a triangle whose side lengths measure 20, 21, and 29 inches respectively, the largest angle is

- (1) acute                      (3) obtuse  
 (2) right                      (4) either acute or obtuse



4. An isosceles triangle has legs of length 10 and a vertex angle that measures  $120^\circ$ . Determine the length of its base in simplest radical form. Draw a diagram illustrating your work.

5. In  $\triangle ABD$  below,  $m\angle B = 65^\circ$ ,  $m\angle D = 48^\circ$ ,  $CD = 15$ ,  $AD = 21$ , and  $AB = 17$ . If points  $B$ ,  $C$ , and  $D$  are collinear find all possible values for  $m\angle ACB$  to the nearest degree.. Hint – First use the Law of Cosines to find  $AC$  then use your result along with the Law of Sines.



## APPLICATIONS

6. A small urban park has a triangular shape formed by the intersection of three roads. The three sides have lengths of 58 yards, 78 yards, and 84 yards. Determine the area of the park to the nearest square yard.

