Name: ___________________________________ Date: _________________

The Area of a Triangle
Algebra 2 with Trigonometry

In this new unit on trigonometry, we will develop formulas that solve a variety of practical applications and involve either the sine or the cosine function. These formulas, though, resemble and are based on the right triangle trigonometry from Algebra 1. Recall:

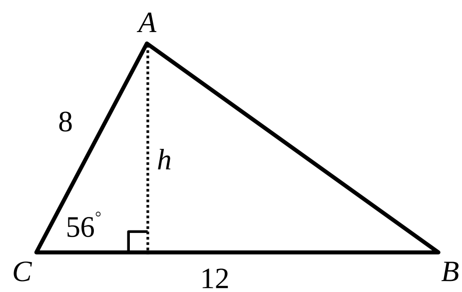
Trigonometric Definitions Based on the Right Triangle

$\sin A = \dfrac{\text{side opposite of } A}{\text{hypotenuse}}$ $\cos A = \dfrac{\text{side adjacent to } A}{\text{hypotenuse}}$ $\tan A = \dfrac{\text{side opposite of } A}{\text{side adjacent to } A}$

Using the sine function, we will now develop a formula for finding the area of a triangle if we know two sides and the measure of the angle included (between) the two sides.

***Exercise* #1:** In ΔABC shown below, $AC = 8$ inches, $BC = 12$ inches and $m\angle C = 56^\circ$.

(a) Determine the height, *h*, of the triangle in terms of $\sin 56^\circ$.



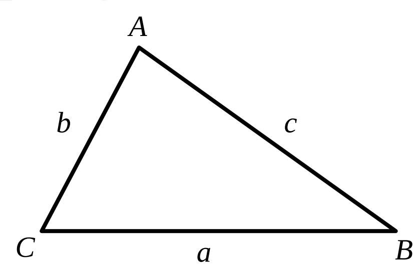
(b) Determine the area of the triangle. Round your answer to the nearest square inch.

This process can be generalized with the following formula:

The Area Formula for a Triangle

For a triangle with side lengths *a*, *b*, and *c*, and opposite angles of *A*, *B*, and *C*, the area is given by:

$$\text{Area} = \frac{1}{2} ab \sin C$$



***Exercise* #2:** Which of the following represents the exact area, in square inches, of an equilateral triangle whose sides have a length of 10 inches?

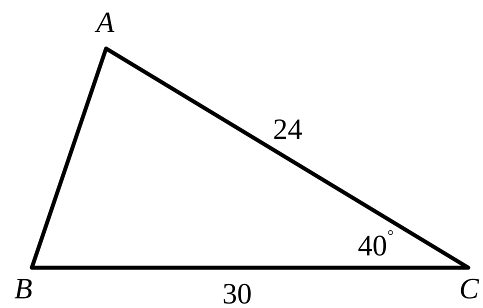
(1) $50\sqrt{2}$ (3) $25\sqrt{2}$

(2) $10\sqrt{3}$ (4) $25\sqrt{3}$ __________

***Exercise* #3:** An isosceles triangle has legs of length 12 inches and base angles that measure 32° each. Find the area of this triangle to the nearest *tenth* of a square inch. Draw a picture to illustrate the triangle.

***Exercise* #4:** In triangle *ABC* shown below, $AC = 24$ cm, $BC = 30$ cm and $m\angle C = 40^{\circ}$.

(a) Determine the area of ΔABC to the nearest square centimeter.



(b) Using your answer from part (a), determine the length of the altitude drawn from *B* to side $\overline{AC}$.

***Exercise* #5:** A garden plot is being designed in the shape of a parallelogram whose sides have lengths of 40 feet and 30 feet. If the acute angles of the parallelogram measure 45°, determine, to the nearest square foot, the area of the garden.

Name: ____________________ Date: ____________

THE AREA OF A TRIANGLE
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. In ΔABC, $AB=10$, $BC=18$, and $m\angle B=68^\circ$. The area of ABC is closest to

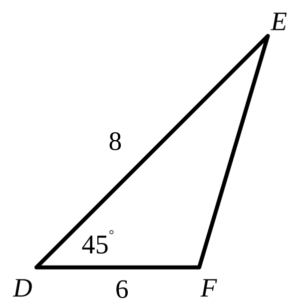
(1) 83 (3) 166

(2) 34 (4) 68 ________

2. Which of the following represents the area of triangle DEF shown below?

(1) 24 (3) 12

(2) $12\sqrt{2}$ (4) $24\sqrt{2}$



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3. An isosceles triangle has legs of length 20 inches and base angles that measure 24°. Which of the following is the area, to the nearest square inch, of this triangle?

(1) 200 (3) 149

(2) 163 (4) 81

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4. Which of the following represents the area of an equilateral triangle whose side lengths measure 8?

(1) 32 (3) $8\sqrt{2}$

(2) $14\sqrt{3}$ (4) $16\sqrt{3}$

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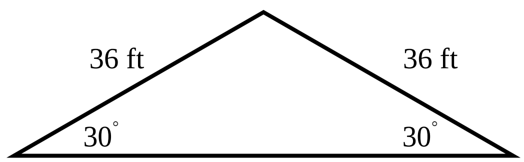
5. A parallelogram whose sides have lengths of 12 feet and 9 feet and whose acute angle measures 52° has an area, accurate to the nearest square foot, of

(1) 43 (3) 85

(2) 58 (4) 67 ________

APPLICATIONS

6. Lanessa is trying to determine the amount of paint she will need for the triangular portion of the front of her house. A gallon of the paint she is using will cover 150 square feet. If the portion she must paint has the shape of an isosceles triangle, shown below, with legs of length 36 feet and base angles of 30°, determine the minimum number of gallons of paint she will need. She can only buy paint in 1-gallon containers.



7. Jeanine would like to put fencing around her flower garden, which has the shape of an equilateral triangle. If Jeanine knows the area of her garden is 90 square feet, determine the length of fencing that Jeanine will need, accurate to the nearest *tenth* of a foot.

REASONING

8. In triangle ABC it is known that $AB = 10$, $AC = 14$ and the area of ABC is equal to 35. Determine the *two* possible values for $m\angle A$ by setting up and solving a trigonometric equation for sine based on the area. Illustrate your two solutions with diagrams of the triangles.