

INVERSE VARIATION
ALGEBRA 2 WITH TRIGONOMETRY

The graphs of rational functions are complex and their analysis is left for higher-level courses. In this lesson, we will explore one type of rational relationship, what is known as inverse variation. As the name implies, when two variables vary inversely, as one increases the other decreases. The first exercise illustrates the mathematics involved.

Exercise #1: When two people sit on a see-saw, in order to balance themselves they must adjust the distance, d , they are away from the fulcrum (balancing point) according to their weights, w . For two people, this takes on the equation:

$$w_1 \cdot d_1 = w_2 \cdot d_2$$

In this formula, w_1 and w_2 represent the weights of the two people and d_1 and d_2 represent their respective distances from the fulcrum.

- (a) How far from the fulcrum must a man who weighs 160 pounds sit if his 50 pound son is sitting 16 feet from the fulcrum?
- (b) If the boy's 80 pound cousin now gets on the see-saw, then how far must the cousin sit from the fulcrum if the boy has not changed his position?

When two variables vary inversely, then **their product always remains constant** throughout a particular problem.

Exercise #2: For each of the following, x and y vary inversely. Determine the missing value.

(a) $x = 2, y = 9$

(b) $x = 3, y = 12$

(c) $x = 8, y = 10$

$x = 6, y = ?$

$x = ?, y = 6$

$x = 2, y = ?$

Exercise #3: If two variables w and z vary inversely then when

- (1) w is increased by 3, z is decreased by 3.
- (2) w is increased by 3, z is increased by 3.
- (3) w is multiplied by 3, z is divided by 3.
- (4) w is multiplied by 3, z is multiplied by 3.



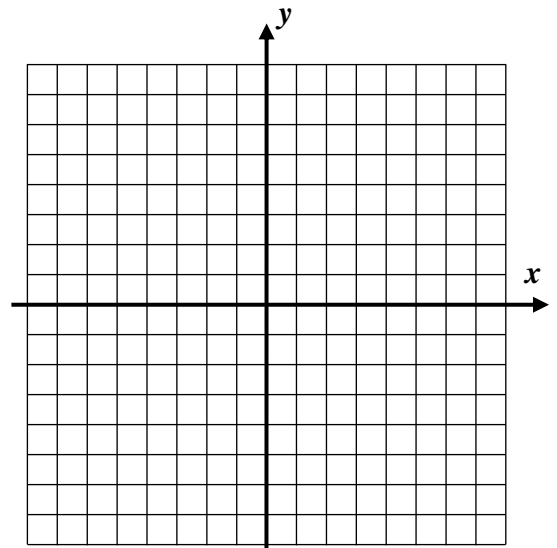
Exercise #4: The force needed to pry a nail from a piece of wood varies inversely with the length of the crowbar used to remove the nail. If a 6 inch long crowbar requires 15 pounds of force, how many pounds are required by an 18 inch long crowbar?

Exercise #5: The amount of time that it takes for Carla to drive home from work each day varies inversely with the speed at which she is driving. If it takes her 45 minutes to drive home in rush hour traffic when she is moving at only 15 miles per hour, *how much time does she save* if she can drive traffic free at 50 miles per hour?

We should also familiarize ourselves with the graph of $xy = k$.

Exercise #6: Consider the equation $xy = 8$.

- (a) Rewrite this equation in function ($y =$) form.
- (b) Using a table of values on your calculator, sketch the graph on the grid given.
- (c) Why is there an error in the table at $x = 0$?



Name: _____

Date: _____

INVERSE VARIATION
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. If x and y vary inversely and $y = 4$ when $x = 5$ then $x = 2$ when $y = ?$

(1) 10

(3) 7

(2) 5

(4) 20

2. If an inverse relationship exists between L and W and $W = 45$ when $L = 20$ then if $L = 60$, $W = ?$

(1) 135

(3) 5

(2) 85

(4) 15

3. In each case below, the two variables given are inversely related. Find the missing value.

(a) $x = 15$, $y = 10$

(b) $w = \frac{1}{2}$, $y = 20$

(c) $a = 50$, $b = 25$

$x = 3$, $y = ?$

$w = ?$, $y = 5$

$a = 10$, $b = ?$

4. Which of the following equations would represent an inverse relationship between x and y ?

(1) $y = 5x$

(3) $y = \frac{10}{x}$

(2) $y = x^2$

(4) $y = 5 - x$

APPLICATIONS

5. If Carlos sits 3 feet from the fulcrum of a see-saw, how far from the fulcrum must his baby-sister who weighs three-times his weight sit?

(1) 1 foot

(3) 3 feet

(2) 6 feet

(4) 9 feet



6. In an ideal gas, if the temperature is held constant, the pressure and volume of a gas will vary inversely. A particular gas has a volume of 120 cubic inches at a pressure of 8 pounds per square. What volume will the gas have, in cubic inches, if the pressure is increased to 32 pounds per square inch?
7. As the weight of a car increases, its fuel economy, the number of miles it can travel per gallon of gasoline, will decrease. These two variable, weight and fuel economy, are inversely related. If a 5000 pound car gets only 12 miles per gallon, what will be the fuel efficiency, in miles per gallon, of a 2000 pound car?
8. A G.I.S. mapping service is trying to determine a relationship between the time, t , it takes for a person to drive from Albany, NY, to Buffalo, NY, and the person's average speed, s . Since this trip is a set distance, these two variables vary inversely. They know it will take a person traveling 50 miles per hour 6 hours to travel from Albany to Buffalo.

(a) Write an equation relating t and s .

(b) Graph this equation on the grid to the right.

