Name: _____ Date: _____

THE DISCRIMINANT OF A QUADRATIC
ALGEBRA 2 WITH TRIGONOMETRY

Since the roots of a quadratic can be found using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, if a, b, and c are all rational numbers, the quantity under the square root, $b^2 - 4ac$, truly dictates what type of numbers the roots of a quadratic (and its x-intercepts) turn out to be. It reduces down to four cases which will be explored in *Exercise* #1.

***Exercise* #1:** For each of the following quadratics, calculate its discriminant, its roots, and state the number and nature (whether they are rational, irrational or imaginary) of the roots.

(a) Case I – The Discriminant is a Perfect Square - $x^2 + 3x - 10 = 0$.

$D = b^2 - 4ac =$ Roots: Number and Nature:

(b) Case II – The Discriminant is Not a Perfect Square - $x^2 - 6x + 7 = 0$.

$D = b^2 - 4ac =$ Roots: Number and Nature:

(c) Case III – The Discriminant is Equal to Zero - $x^2 - 10x + 25 = 0$.

$D = b^2 - 4ac =$ Roots: Number and Nature:

(d) Case IV – The Discriminant is Less than Zero - $x^2 - 8x + 20 = 0$

$D = b^2 - 4ac =$ Roots: Number and Nature:

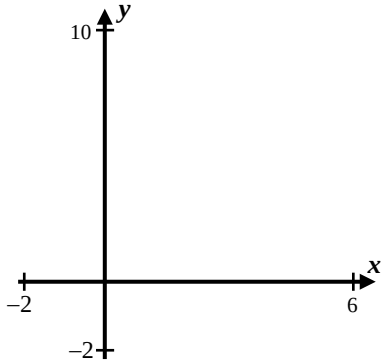
In the last lesson, we explored Case IV extensively. In the case where the discriminant is negative, the roots of the quadratic are **imaginary** and it does not have *x*-intercepts (i.e. it does not cross the *x*-axis).

***Exercise* #2:** By using only the discriminant, determine the number and nature of the roots of each of the following quadratics.

(a) $2x^2 + 7x - 4 = 0$ (b) $x^2 - 8x + 25 = 0$ (c) $4x^2 + 4x + 1 = 0$

(d) $x^2 + 6x + 15 = 0$ (e) $4x^2 - 4x - 7 = 0$ (f) $3x^2 - 7x + 2 = 0$

***Exercise* #3:** Consider the quadratic function whose equation is $y = x^2 - 4x + 4$. Determine the number of *x*-intercepts of this quadratic from the value of its discriminant. Then, sketch its graph on the axes given. We say that this parabola is **tangent** to the *x*-axis.



***Exercise* #4:** Which of the following parabolas has two unequal, rational *x*-intercepts?

(1) $y = x^2 - 2x - 1$ (3) $y = x^2 - 8x + 16$

(2) $y = x^2 + 2x - 15$ (4) $y = x^2 - 3x + 5$ _____

***Exercise* #5:** For what values of *a* will the parabola $y = ax^2 + 4x + 2$ not cross the *x*-axis?

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THE DISCRIMINANT OF A QUADRATIC
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. For each of the following quadratic equations, determine the number and the nature of the roots by first calculating the quadratic's discriminant.

   (a) $2x^2 + 4x + 5 = 0$ (b) $9x^2 - 12x + 4 = 0$ (c) $4x^2 - 13x + 3 = 0$

   (d) $x^2 + 8x + 11 = 0$ (e) $4x^2 + 4x - 7 = 0$ (f) $36x^2 - 12x + 1 = 0$

   (g) $-3x^2 + 4x - 8 = 0$ (h) $3x^2 + 8x + 4 = 0$ (i) $x^2 + 8x + 41 = 0$

2. The roots of $x^2 + 4x - 7 = 0$ are

   (1) unequal and rational (3) unequal and irrational

   (2) unequal and imaginary (4) equal and rational _____

3. Which of the following quadratics has imaginary roots?

   (1) $x^2 + 3x - 5 = 0$ (3) $2x^2 - 3x + 1 = 0$

   (2) $x^2 + 6x + 10 = 0$ (4) $x^2 - 7x + 10 = 0$ _____

4. Which of the following quadratic, when graphed, would touch the *x*-axis exactly once?

   (1) $y = x^2 - 2x - 3$ (3) $y = x^2 + 5x - 2$

   (2) $y = 2x^2 + 3x + 7$ (4) $y = x^2 - 12x + 36$ _____

5. If graphed, which of the following parabolas would lie entirely below the *x*-axis?

(1) $y = x^2 + 5x + 10$ (3) $y = -2x^2 + 6x - 5$

(2) $y = -2x^2 - 5x + 3$ (4) $y = x^2 + 6x + 9$

_____

6. Which parabola below, when graphed, would cross the *x*-axis at two unequal irrational locations?

(1) $y = 2x^2 + 11x + 12$ (3) $y = 9x^2 - 6x + 1$

(2) $y = x^2 + 2x - 4$ (4) $y = 2x^2 + 4x + 9$

_____

REASONING

7. Determine all values of *a* that will cause the parabola $y = ax^2 + 10x + 1$ to cross the *x*-axis at two distinct locations.

8. Consider the parabola whose equation is $y = x^2 - 4x$ and the line whose equation is $y = 2x + b$, where *b* is some unknown constant. Determine the value of *b* such that the line and the parabola will intersect at exactly one location. Then, sketch the system of equations on the axes below. Label their intersection point.