Name: ___________________________________ Date: _________________

SOLVING QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS
ALGEBRA 2 WITH TRIGONOMETRY

As we saw in the last unit, the roots or zeros of any quadratic equation can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since this formula contains a square root, it is fair to investigate solutions to quadratic equations now when the quantity $b^2 - 4ac$, known as the **discriminant**, is negative.

***Exercise* #1:** Use the quadratic formula to find all solutions to the following equation. Express your answers in simplest $a + bi$ form. Check your answers by using the **STORE** feature on your calculator.

$$x^2 - 4x + 29 = 0$$

As long as our solutions can include complex numbers, then any quadratic equation can be solved for two roots. We have actually seen quadratics already that have had complex roots in Lesson #1 of this unit.

***Exercise* #2:** Solve each of the following quadratic equations. Express your answers in simplest $a + bi$ form. Check your answers by using the **STORE** feature on your calculator.

(a) $x^2 - 5x + 30 = 7x - 10$

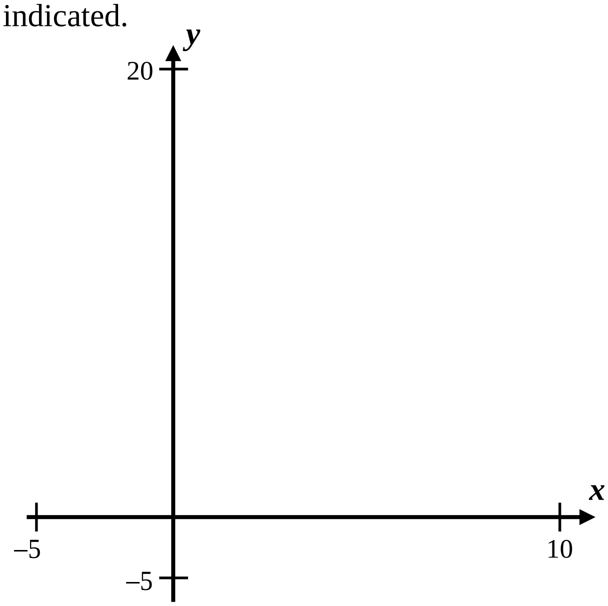
(b) $x^2 + 16x + 15 = 10x + 4$

There is an interesting connection between the x-intercepts of a parabola and complex roots with non-zero imaginary parts. This will be explored in the next lesson in more detail. But, the next exercise illustrates an important concept.

***Exercise* #3:** Consider the parabola whose equation is $y = x^2 - 6x + 13$.

(a) Algebraically find the x-intercepts of this parabola. Express your answers in simplest $a + bi$ form.

(b) Using your calculator, sketch a graph of the parabola on the axes below. Use the window indicated.



(c) From your answers to (a) and (b), what can be said about parabolas whose zeros are complex roots with non-zero imaginary parts?

***Exercise* #4:** Use the discriminant of each of the following quadratics to determine whether it has x-intercepts.

(a) $y = x^2 - 3x - 10$

(b) $y = x^2 + 6x + 10$

(c) $y = 2x^2 + 3x + 5$

***Exercise* #5:** Which of the following quadratic functions, when graphed, would not cross the x-axis?

(1) $y = 2x^2 + 5x - 3$

(2) $y = -x^2 - x + 6$

(3) $y = 4x^2 - 4x + 5$

(4) $y = 3x^2 - 13x + 4$

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Name: ___________________________________ Date: _________________

SOLVING QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Solve each of the following quadratic equations. Express your solutions in simplest $a+bi$ form. Check.

(a) $x^2+4x+20=12x-5$

(b) $x^2=x-1$

(c) $2x^2-25x+27=-15x-10$

(d) $8x^2+36x+24=12x+5$

(e) $x^2+6x+15=8x-2$

(f) $4x^2+38x+50=10x-35$

2. Which of the following represents the solution set to the equation $x^2 - 2x + 2 = 0$?

(1) $x = -1$ or 2 (3) $x = 2 \pm i$

(2) $x = 1 \pm 2i$ (4) $x = 1 \pm i$

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3. The solutions to the equation $x^2 + 6x + 11 = 0$ are

(1) $x = -3 \pm i\sqrt{2}$ (3) $x = -6 \pm i\sqrt{11}$

(2) $x = -3 \pm 2i\sqrt{2}$ (4) $x = -6 \pm 2i\sqrt{11}$

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4. Using the determinant, $b^2 - 4ac$, determine whether each of the following quadratics has real or imaginary zeros.

(a) $y = 2x^2 - 7x + 6$ (b) $y = 3x^2 + 2x + 1$ (c) $y = x^2 - 8x + 14$

(d) $y = 2x^2 - 12x + 26$ (e) $y = -2x^2 + 6x - 5$ (f) $y = 4x^2 - 4x + 1$

5. Which of the following quadratics, if graphed, would lie entirely above the *x*-axis. Try to use the discriminant to solve this problem and then graph to check.

(1) $y = 2x^2 + x - 21$ (3) $y = x^2 - 4x + 7$

(2) $y = x^2 - x - 6$ (4) $y = x^2 - 10x + 16$

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REASONING

6. For what values of c will the quadratic $y = x^2 + 6x + c$ have no real zeros? Set up and solve an inequality for this problem.