

Name: _____

Date: _____

MULTIPLICATION AND DIVISION OF COMPLEX NUMBERS ALGEBRA 2 WITH TRIGONOMETRY

The set of complex numbers is **closed** under the operations of multiplication and division (except by zero). What this means is that when two complex numbers are multiplied, their product is another complex number - similarly for division. Multiplication is easy enough because it follows the basic principle of multiplying two binomials.

Exercise #1: Find the following products. Write each of your answers as a complex number in the form $a + bi$.

(a) $(3 + 5i)(7 + 2i)$

(b) $(-2 + 6i)(3 - 2i)$

(c) $(4 + i)(-5 - 3i)$

A curious result occurs when multiplying two numbers known as **complex conjugates**. The numbers $(a + bi)$ and $(a - bi)$ are the general form of complex conjugates.

Exercise #2: Find each of the following products of complex conjugates.

(a) $(4 + 2i)(4 - 2i)$

(b) $(-5 + i)(-5 - i)$

(c) $(6 + 3i)(6 - 3i)$

Exercise #3: What is true about each of the products that you found in *Exercise #2*?

Exercise #4: Show that the product of a complex number $a + bi$ with its conjugate is always a purely real number and that number is equal to $a^2 + b^2$.



Dividing complex numbers is not as straightforward as multiplying them. It is not at all obvious why two numbers of the form $a + bi$ should divide to be another of this form. The key is in the conjugate multiplication with which we just worked.

Exercise #4: Consider the division given by $\frac{3+2i}{4-5i}$.

(a) If division of complex numbers is consistent with division of real number then

(b) Write the original quotient in simplest $a + bi$ form by multiplying it by the fraction in (a).

$$\frac{4+5i}{4+5i} =$$

Exercise #5: Find each of the following quotients in simplest $a + bi$ form.

(a) $\frac{2-4i}{5-3i}$

(b) $\frac{22+3i}{4-10i}$

(c) $\frac{10-5i}{3+4i}$

Exercise #6: Which of the following represents the reciprocal of $3+9i$?

(1) $\frac{1}{3} + \frac{1}{9}i$

(3) $\frac{1}{30} - \frac{1}{10}i$

(2) $\frac{1}{9} - \frac{1}{3}i$

(4) $\frac{1}{10} + \frac{1}{3}i$



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MULTIPLICATION AND DIVISION OF COMPLEX NUMBERS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Find each of the following products in simplest $a + bi$ form.

(a) $(5 - 2i)(-1 + 7i)$

(b) $(3 + 9i)(2 + 4i)$

(c) $(-4 - i)(-2 + 6i)$

2. If the product of $(2 + 7i)$ and $(5 + 3i)$ were plotted, in which of the following quadrants would it lie?

(1) I

(3) III

(2) II

(4) IV

3. Multiply each of the following conjugate pairs by using the fact that $(a + bi)(a - bi) = a^2 + b^2$.

(a) $(5 - 7i)(5 + 7i)$

(b) $(10 + i)(10 - i)$

(c) $(-3 + 8i)(-3 - 8i)$

4. The product of $(-8 + 2i)$ and its conjugate is equal to

(1) $64 + 4i$

(3) 68

(2) 60

(4) $60 - 4i$

5. The complex computation $(6 + 2i)(6 - 2i) - (3 - 4i)(3 + 4i)$ can be simplified to

(1) 15

(3) -10

(2) 39

(4) -35



6. Write each of the following quotients in simplest $a + bi$ form.

(a) $\frac{5+10i}{6-2i}$

(b) $\frac{11+2i}{3-4i}$

(c) $\frac{3+4i}{9-3i}$

(d) $\frac{2-4i}{2+4i}$

(e) $\frac{9-17i}{6+2i}$

(f) $\frac{26-2i}{12+3i}$

7. Find the reciprocal (multiplicative inverse) of each of the following complex numbers. Express your answers in simplest $a + bi$ form.

(a) $4 - 2i$

(b) $1 + i$

(c) $\frac{3+4i}{5}$

8. Which of the following is equivalent to $\frac{7-i}{3+i}$?

(1) $2 - i$

(3) $\frac{7}{3} + i$

(2) $\frac{11}{5} - i$

(4) $\frac{7}{3} - i$

