Name: ___________________________________ Date: _________________

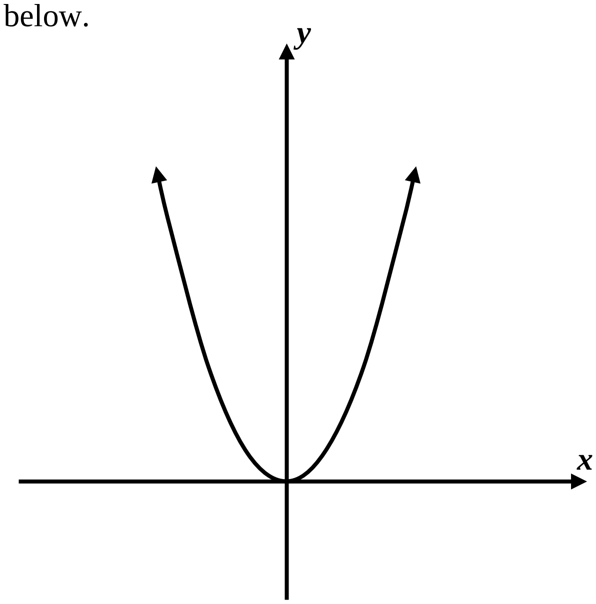
IMAGINARY NUMBERS
ALGEBRA 2 WITH TRIGONOMETRY

Recall that in the Real Number System, it is not possible to take the square root of a negative quantity because whenever a real number is squared it is non-negative. This fact has a ramification for finding the *x*-intercepts of a parabola, as *Exercise* #1 will illustrate.

***Exercise* #1:** On the axes below, a sketch of $y = x^2$ is shown. Now, consider the parabola whose equation is given in function notation as $f(x) = x^2 + 1$.

(a) How is the graph of $y = x^2$ shifted to produce the graph of $f(x)$?

(b) Create a quick sketch of $f(x)$ on the axes below.



(c) What can be said about the *x*-intercepts of the function $y = f(x)$?

(d) Algebraically, show that these intercepts do not exist, in the Real Number System, by solving the incomplete quadratic $x^2 + 1 = 0$.

Since we cannot solve this equation using Real Numbers, we introduce a new number, called *i*, the basis of imaginary numbers. Its definition allows us to now have a result when finding the square root of a negative real number. Its definition is given below.

THE DEFINITION OF THE IMAGINARY NUMBER *i*

$$i = \sqrt{-1}$$

***Exercise* #2:** Simplify each of the following square roots in terms of *i*.

(a) $\sqrt{-9}$ (b) $\sqrt{-100}$ (c) $\sqrt{-32}$ (d) $\sqrt{-18}$

***Exercise* #2:** Solve each of the following incomplete quadratics. Place your answers in simplest radical form.

(a) $5x^2+8=-12$ (b) $\frac{1}{2}x^2+20=2$ (c) $2x^2-10=-36$

***Exercise* #3:** Which of the following is equivalent to $5i\cdot 6i$?

(1) $30i$ (3) -30

(2) $11i$ (4) -11

Powers of *i* display a remarkable pattern that allow us to simplify large powers of *i* into one of 4 cases. This pattern is discovered in *Exercise* #4.

***Exercise* #4:** Simplify each of the following powers of *i*.

$i^1=i$ $i^2=$ $i^3=$ $i^4=$

$i^5=$ $i^6=$ $i^7=$ $i^8=$

We see, then, from this pattern that every power of *i* is either $-1, 1, i,$ or $-i$. And the pattern will repeat.

***Exercise* #5:** From the pattern of *Exercise* #4, simplify each of the following powers of *i*.

(a) $i^{38}=$ (b) $i^{21}=$ (c) $i^{83}=$ (d) $i^{40}=$

***Exercise* #6:** Which of the following is equivalent to $5i^{16}+3i^{23}+i^{26}$?

(1) $8+2i$ (3) $5-4i$

(2) $4-3i$ (4) $2+7i$

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IMAGINARY NUMBERS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. The imaginary number i is defined as

(1) -1 (3) $\sqrt{-4}$

(2) $\sqrt{-1}$ (4) $(-1)^2$ __________

2. Which of the following is equivalent to $\sqrt{-128}$?

(1) $8\sqrt{2}$ (3) $-8\sqrt{2}$

(2) $8i$ (4) $8i\sqrt{2}$ __________

3. The sum $\sqrt{-9}+\sqrt{-16}$ is equal to

(1) 5 (3) $7i$

(2) $5i$ (4) 7 __________

4. Which of the following powers of i is *not* equal to one?

(1) i^{16} (3) i^{32}

(2) i^{26} (4) i^{48} __________

5. Which of the following represents all solutions to the equation $\frac{1}{3}x^2+10=7$?

(1) $x=\pm 3i$ (3) $x=\pm i$

(2) $x=\pm 5i$ (4) $x=\pm 2i$ __________

6. Solve each of the following incomplete quadratics. Express your answers in simplest radical form.

(a) $2x^2+100=-62$

(b) $\frac{2}{3}x^2+20=2$

7. Which of the following represents the solution set of $\frac{1}{2}x^2 - 12 = -37$?

(1) $\pm 7i$ (3) $\pm 5i\sqrt{2}$

(2) $\pm 7i\sqrt{2}$ (4) $\pm 3i\sqrt{2}$

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8. Simplify each of the following powers of i into either $-1, 1, i,$ or $-i$.

(a) i^2 (b) i^3 (c) i^4 (d) i^{11}

(e) i^{41} (f) i^{30} (g) i^{25} (h) i^{36}

(i) i^{51} (j) i^{45} (k) i^{80} (l) i^{70}

9. Which of the following is equivalent to $i^7 + i^8 + i^9 + i^{10}$?

(1) 1 (3) $1 - i$

(2) $2 + i$ (4) 0

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10. When simplified the sum $5i^{18} + 7i^{25} + 2i^{28} + 6i^{43}$ is equal to

(1) $2 - 4i$ (3) $5 - 7i$

(2) $-3 + i$ (4) $8 + i$

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11. The product $(6 + 2i)(4 - 3i)$ can be written as

(1) $24 - 6i$ (3) $2 + 5i$

(2) $18 + 10i$ (4) $30 - 10i$

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