Name: ____________________________________ Date: _________________

FRACTIONAL POWERS AND HIGHER ORDER ROOTS
ALGEBRA 2 WITH TRIGONOMETRY

As strange as it may sound, fractional exponents exist and are quite important in higher-level mathematics. Just as negative and zero exponents were foreign when you first visited them, fractional powers will also seem strange. The key to understanding them is based on recalling the following fundamental exponent law:

$$\left(x^a\right)^b = x^{a\cdot b}$$

The first exercise will help you understand the most basic fractional exponent, $x^{1/2}$.

***Exercise* #1:** Answer each of the following questions.

(a) Based on exponent laws, what must $\left(x^{1/2}\right)^2$ be equal to?

(b) Based on our experience with inverses what is $\left(\sqrt{x}\right)^2$ equal to?

(c) Based on your answer to parts (a) and (b) what is $x^{1/2}$ equal to?

(d) Experiment with this on your calculator by evaluating:

$9^{1/2} =$ and $100^{1/2} =$

The same type of reasoning could be used to establish the basic facts about fractional powers and roots.

FRACTIONAL POWERS AND ROOTS – OBSERVATION #1

For any positive integer *a*, $x^{1/a} = \sqrt[a]{x}$

***Exercise* #2:** Evaluate each of the following without the use of your calculator. Only use it if absolutely necessary.

(a) $8^{1/3}$ (b) $81^{1/4}$ (c) $125^{-1/3}$ (d) $256^{-1/4}$

***Exercise* #3:** Which of the following is equivalent to $x^{-1/5}$?

(1) $\dfrac{1}{5\sqrt{x}}$ (3) $-5\sqrt{x}$

(2) $\dfrac{1}{\sqrt[5]{x}}$ (4) $-\sqrt[5]{x}$

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Fractional powers can have numerators other than 1. When this occurs, we simply have a combination of our normal powers along with taking roots. This is illustrated in the next exercise.

***Exercise* #4:** Consider the expression $x^{4/3}$.

(a) Write this expression in the form $\left(x^{1/a}\right)^{b}$ where a and b are integers.

(b) Write this expression in the form $\left(x^{b}\right)^{1/a}$ where a and b are the same integers as in part (a).

(c) Write $x^{4/3}$ as a combination of a root and a power in two different ways based on your answers from parts (a) and (b).

(d) Evaluate $8^{4/3}$ without the use of your calculator.

FRACTIONAL POWERS AND ROOTS – OBSERVATION #2

For any two positive integers a and b, $x^{b/a}=\sqrt[a]{x^{b}}$ and $x^{b/a}=\left(\sqrt[a]{x}\right)^{b}$

***Exercise* #5:** Evaluate each of the following without the use of your calculator.

(a) $27^{2/3}$ (b) $4^{5/2}$ (c) $16^{-3/4}$

***Exercise* #6:** Which of the following is equivalent to $(8x)^{2/3}$?

(1) $4\sqrt[3]{x^{2}}$ (3) $2\sqrt{x^{3}}$

(2) $\sqrt[3]{8x^{2}}$ (4) $32\sqrt{x^{3}}$

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FRACTIONAL POWERS AND HIGHER ORDER ROOTS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Rewrite the following as equivalent roots and then evaluate as many as possible **without your calculator.**

(a) $36^{1/2}$ (b) $27^{1/3}$ (c) $32^{1/5}$ (d) $100^{-1/2}$

(e) $625^{1/4}$ (f) $49^{1/2}$ (g) $81^{-1/4}$ (h) $343^{1/3}$

2. Evaluate each of the following by considering the root and power indicated by the exponent. Do as many as possible **without your calculator**.

(a) $8^{2/3}$ (b) $4^{3/2}$ (c) $16^{3/4}$ (d) $81^{5/4}$

(e) $4^{-5/2}$ (f) $128^{3/7}$ (g) $625^{3/4}$ (h) $243^{3/5}$

3. Which of the following is equivalent to $x^{-1/2}$?

(1) $-\frac{1}{2}x$ (3) $\frac{1}{\sqrt{x}}$

(2) $-\sqrt{x}$ (4) $-\frac{1}{2x}$

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4. The expression $y^{5/2}$ can be equivalently written as

(1) $\left(\sqrt{y}\right)^5$ (3) $\sqrt[5]{y}$

(2) $\left(\sqrt[5]{y}\right)^2$ (4) $5\sqrt{y}$ __________

5. Written without fractional or negative exponents, $x^{-3/2}$ is equal to

(1) $-\frac{3x}{2}$ (3) $\frac{1}{\sqrt{x^3}}$

(2) $\frac{1}{\sqrt[3]{x^2}}$ (4) $-\frac{1}{\sqrt{x}}$ __________

6. The monomial $(4x)^{5/2}$ can be rewritten equivalently as

(1) $\sqrt{4x^5}$ (3) $\sqrt[5]{4x^2}$

(2) $2\sqrt{x^5}$ (4) $32\sqrt{x^5}$ __________

7. Expressed in an equivalent manner, $(8x)^{-1/3}$ is

(1) $\frac{1}{2\sqrt[3]{x}}$ (3) $\frac{1}{8\sqrt{x}}$

(2) $\frac{\sqrt{x}}{2}$ (4) $\frac{1}{\sqrt[3]{2x}}$ __________

Reasoning

8. Equations such as the one below are solved similar to solving incomplete quadratics. Solve the following equation for the value of x. As a hint, the inverse operation to raising a quantity to the $3/2$ consists of raising it to the $2/3$.

$$2x^{3/2}+6=60$$