Name: _____ Date: _____

POWER FUNCTIONS AND REGRESSION
ALGEBRA 2 WITH TRIGONOMETRY

Now that we have exponents of all types (except irrational), we can introduce the concept of a **power function.**

POWER FUNCTIONS

For any real-valued constants *a* and *b*, a function that can be placed in the form:

$$f(x) = ax^b$$

***Exercise* #1:** Determine whether each of the following is a power function and, if it is, place it in the form $y = ax^b$ if it is not already in such a form.

(a) $y = 5x^3$ (b) $y = 5x - 4$ (c) $y = \dfrac{\sqrt[3]{x}}{4}$ (d) $y = \dfrac{10}{x^2}$

Power functions have simple forms and share many common characteristics.

***Exercise* #2:** Consider a power function with the general form $y = ax^b$.

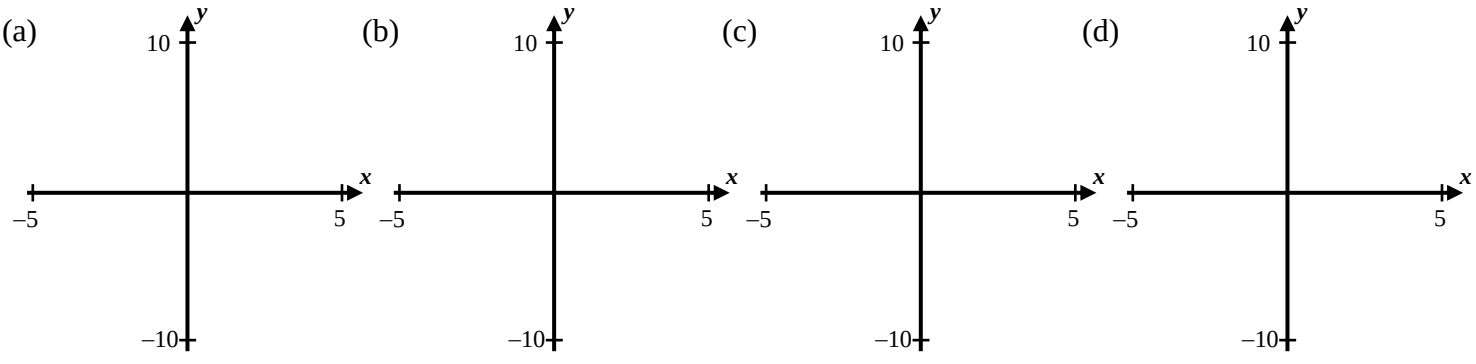
(a) What is the *y*-intercept of any power function where $b > 0$? Explain or justify your answer.

(b) What is true about the *y*-intercept of any power function where $b < 0$? Cite specific examples to illustrate your answer.

The behavior of a power function is highly dependent on its exponent. Similarities exist between odd positive powers and even positive powers.

***Exercise* #3:** Create a sketch of each of the following simple power functions using the windows indicated.

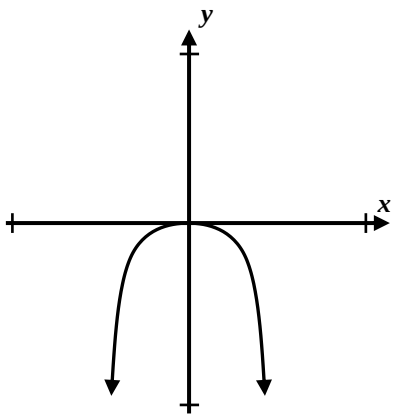
(a) $y = x^2$ (b) $y = x^3$ (c) $y = x^4$ (d) $y = x^5$



***Exercise* #4:** Which of the following power functions is shown in the graph below? Explain your choice.

(1) $y = -4x^7$ (3) $y = 6x^8$

(2) $y = -3x^{10}$ (4) $y = 5x^9$



_____

Explanation:

Power functions are commonly used to fit data through the process of **power regression**. This procedure is identical to that of **linear regression**, except that we will fit the data with an equation of the form $y = ax^b$ instead of one of the form $y = ax + b$. Power regressions are most commonly used in situations where the data must include the point $(0, 0)$, in other words the two variables must be zero at the same time.

***Exercise* #5:** Kirk was measuring the height of germinating corn plants as a function of the days since they first broke the soil for his 8th grade science fair project. He measured the following data:

Time Since Germination (days)	2	5	14	20	25	35	42
Average Plant Height (cm)	6.3	9.1	15.8	19.1	20.6	24.7	27.6

(a) Use your calculator to find an equation of the form $y = ax^b$ that best models this data set. Round both constants to the nearest *hundredth*.

(b) Use your model in part (a) to **interpolate** the height of the corn on the 30th day. Round your height to the nearest *tenth* of a centimeter.

(c) Use your model from part (a) to **extrapolate** the height of the corn at the end of the 60th day. Round your height to the nearest *tenth* of a centimeter.

(d) For simplicity, Kirk concluded that the average plant height can be modeled with an equation of the form $y = a\sqrt{x}$. Is he justified? Explain.

Name: _____ Date: _____

POWER FUNCTIONS AND REGRESSION
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Which of the following is *not* an example of a power function?

   (1) $y = 5x^4$ (3) $y = \frac{6}{x^3}$

   (2) $y = x^3 - 1$ (4) $y = \frac{1}{\sqrt{x}}$ _____

2. Which of the following power functions has no *y*-intercept?

   (1) $y = 4x^2$ (3) $y = 6x^{-3}$

   (2) $y = 5x^{1/4}$ (4) $y = x$ _____

3. If the power function $y = \frac{\sqrt[5]{x}}{2}$ was placed in the form $y = ax^b$ then which of the following would be true?

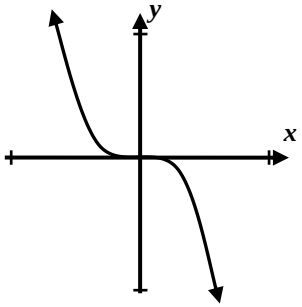
   (1) $a = 2$ and $b = -5$ (3) $a = \frac{1}{2}$ and $b = 1/5$

   (2) $a = 2$ and $b = 1/5$ (4) $a = \frac{1}{5}$ and $b = -5$ _____

4. Which of the following power functions is shown graphed below?

   (1) $y = -6x^4$ (3) $y = 7x^3$

   (2) $y = 2x^8$ (4) $y = -3x^5$ _____



5. If the point $(8, -12)$ lies on the power function whose equation is $y = ax^{1/3}$ then the value of *a* must be

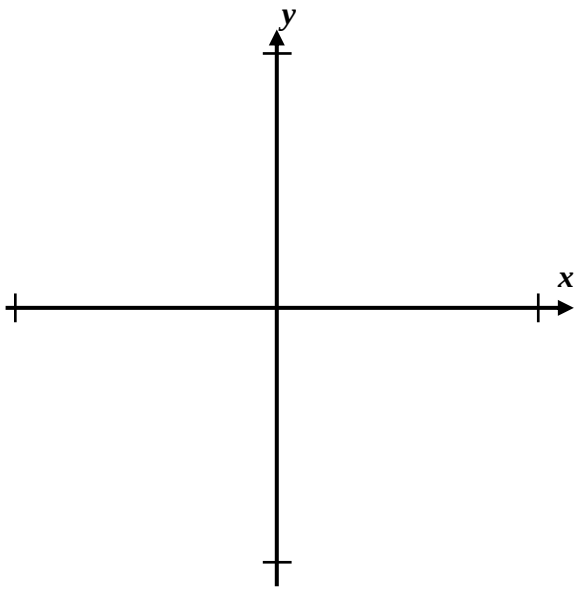
   (1) −6 (3) 8

   (2) −4 (4) 24 _____

6. Consider the system of power equations shown below:

$$y = 2x^3 \text{ and } y = 18x$$

(a) Solve this system graphically. Show your solution on the axes below. Be sure to label your curves and your window.



(b) Solve this system algebraically by setting the equations equal and then using the Zero Product Law.

APPLICATIONS

7. A social networking website was recently launched. The number of its members was measured compared to the number of days since it was launched.

Days Since Launch	2	7	10	14	21	27
Number of Members	40	275	410	768	1250	2500

(a) Find a power function, of the form $y = ax^b$, that best fits this data. Round your constants to the nearest *hundredth*.

(b) Use your model from part (a) to estimate the number of members 100 days after the site was launched. Round your answer to the nearest integer.

(c) Is the calculation you made in part (a) an example of interpolation or extrapolation? Explain.

(d) The networking site would like to model their membership with a function of the form $y = a\sqrt{x^3}$. Does the regression model from (a) support their model? Explain.