Name: _____ Date: _____

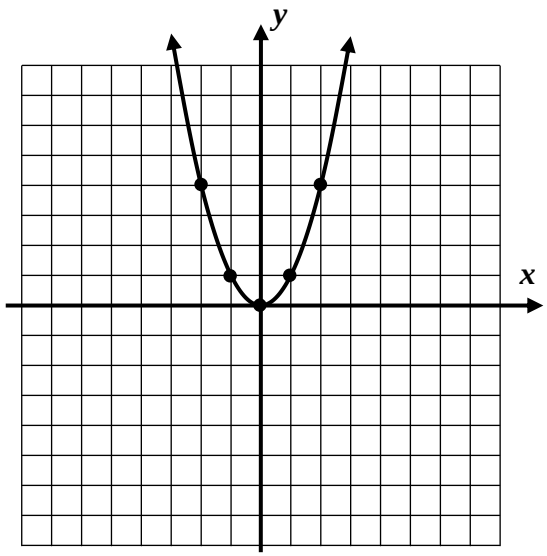
SHIFTING PARABOLAS AND COMPLETING THE SQUARE
ALGEBRA 2 WITH TRIGONOMETRY

Parabolas, and graphs more generally, can be moved horizontally and vertically by simple manipulations of their equations. This is known as **shifting** or **translating** a graph. These manipulations are illustrated in the first exercise.

***Exercise* #1:** The function $y = x^2$ is shown already graphed on the grid below. For parts (a) and (b), graph the function given and describe how the graph of $y = x^2$ has been shifted in order to produce the new graph.

(a) $y = (x-4)^2 + 2$

(b) $y = (x+3)^2 - 5$



***Exercise* #2:** If the parabola $y = x^2$ were shifted 6 units left and 2 units down, its resulting equation would be which of the following? Verify by graphing your answer and seeing if its turning point is at $(-6, -2)$.

(1) $y = (x+6)^2 + 2$ (3) $y = (x-6)^2 - 2$

(2) $y = (x+6)^2 - 2$ (4) $y = (x-6)^2 + 2$

_____

***Exercise* #3:** Which of the following represents the turning point of the function $f(x) = (x-8)^2 - 4$?

(1) $(-8, -4)$ (3) $(-8, 4)$

(2) $(8, 4)$ (4) $(8, -4)$

_____

Every parabola can be placed in what is known as its **vertex form**. For simple parabolas that are shifts of the function $y = x^2$, this will always take on the following form.

THE VERTEX FORM OF A PARABOLA

$$y = (x-h)^2 + k$$

where (h, k) is the vertex or turning point of the parabola.

It is possible to place every quadratic function of the form $y = x^2 + bx + c$ in the above form by a process known as **completing the square**. The following exercise will illustrate the technique.

***Exercise* #4:** Consider the quadratic $y = x^2 + 8x + 7$.

(a) Write this quadratic function in the form $y = (x-h)^2 + k$ by completing the square.

(b) Identify the turning point of the parabola from your answer in part (a). Verify your answer using tables on your calculator.

Although very algorithmic in nature, the technique of completing the square arises often in mathematics and is extremely useful in determining the turning point of a parabola.

***Exercise* #5:** Which of the following is equivalent to $y = x^2 - 6x + 2$?

(1) $y = (x-3)^2 - 7$ (3) $y = (x+3)^2 - 2$

(2) $y = (x-3)^2 + 2$ (4) $y = (x-6)^2 - 10$

***Exercise* #6:** Consider the quadratic function $f(x) = x^2 - 2x - 8$.

(a) Write the function in its vertex form and identify the coordinates of the parabola's turning point.

(b) Find its x-intercepts algebraically by setting the equation you found in part (a) to zero.

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SHIFTING PARABOLAS AND COMPLETING THE SQUARE
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Which of the following equations would result from shifting $y = x^2$ five units right and four units up?

   (1) $y = (x-5)^2 + 4$ (3) $y = (x-4)^2 - 5$

   (2) $y = (x+5)^2 + 4$ (4) $y = (x+4)^2 - 5$ _____

2. Which of the following represents the turning point of the parabola whose equation is $y = (x+3)^2 - 7$?

   (1) $(3, -7)$ (3) $(-7, -3)$

   (2) $(-3, 7)$ (4) $(-3, -7)$ _____

3. Which of the following quadratic functions would have a turning point at $(6, -2)$?

   (1) $y = (x+6)^2 - 2$ (3) $y = (x-6)^2 - 2$

   (2) $y = (x+2)^2 + 6$ (4) $y = (x-2)^2 - 6$ _____

4. Which of the following is equivalent to $y = x^2 + 12x - 4$?

   (1) $y = (x+6)^2 - 40$ (3) $y = (x+6)^2 - 4$

   (2) $y = (x-6)^2 + 28$ (4) $y = (x+12)^2 - 148$ _____

5. In vertex form, the parabola $y = x^2 - 10x + 8$ would be written as

   (1) $y = (x-5)^2 - 33$ (3) $y = (x-10)^2 - 92$

   (2) $y = (x-5)^2 - 17$ (4) $y = (x-10)^2 - 108$ _____

6. The turning point of the parabola $y = x^2 + 5x - 2$ is

   (1) $(2.5, 12.75)$ (3) $(-2.5, -8.25)$

   (2) $(-5, -10.5)$ (4) $(-2.5, -17.5)$ _____

7. Write each of the following quadratic functions in its vertex form by completing the square.

(a) $y = x^2 + 12x + 50$

(b) $y = x^2 - 10x + 7$

8. Consider the quadratic function whose equation is $y = x^2 + 6x - 40$.

(a) Determine the *y*-intercept of this function algebraically.

(b) Write the function in its vertex form. State the coordinates of its turning point.

(c) Algebraically find the *x*-intercepts of the function by setting the equation that you found in part (b) equal to zero.

(d) Sketch a graph of the parabola, showing all relevant features found in parts (a) through (c).

9. The quadratic function shown graphed to the right has the form $y = x^2 + bx + c$. Determine its equation first in vertex form and then determine the values of *b* and *c*.