

## INVERSES OF LINEAR FUNCTIONS ALGEBRA 2 WITH TRIGONOMETRY

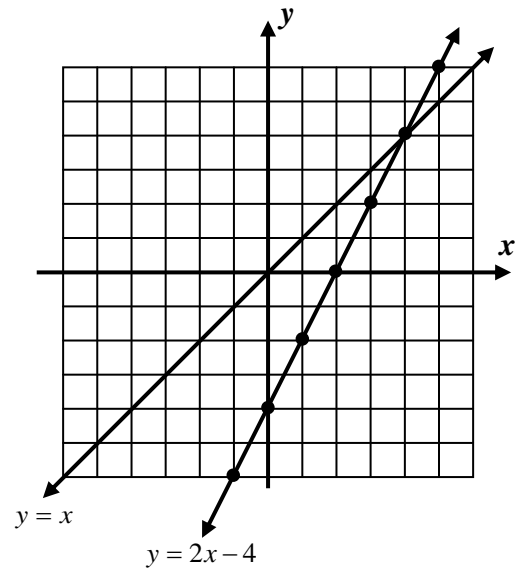
Recall that functions have inverses that are also functions if they are one-to-one. With the exception of horizontal lines, all linear functions are one-to-one and thus have inverses that are also functions. In this lesson we will investigate these inverses and how to find their equations.

**Exercise #1:** On the grid below the linear function  $y = 2x - 4$  is graphed along with the line  $y = x$ .

(a) How can you quickly tell that  $y = 2x - 4$  is a one-to-one function?

(b) Graph the inverse of  $y = 2x - 4$  on the same grid. Recall that this is easily done by switching the  $x$  and  $y$  coordinates of the original line.

(c) What can be said about the graphs of  $y = 2x - 4$  and its inverse with respect to the line  $y = x$ ?



(d) Find the equation of the inverse in  $y = mx + b$  form.

(e) Find the equation of the inverse in  $y = \frac{x+b}{a}$  form.

As we can see from part (e) in *Exercise #1*, inverses of linear functions include the inverse operations of the original function but in reverse order. This gives rise to a simple method of finding the equation of any inverse. **Simply switch the  $x$  and  $y$  variables in the original equation and solve for  $y$ .**

**Exercise #2:** Which of the following represents the equation of the inverse of  $y = 5x - 20$ ?

(1)  $y = -\frac{1}{5}x + 20$

(3)  $y = \frac{1}{5}x - 4$

(2)  $y = \frac{1}{5}x - 20$

(4)  $y = \frac{1}{5}x + 4$



Although this is a simple enough procedure, certain problems can lead to common errors when solving for  $y$ . Care should be taken with each algebraic step.

**Exercise #3:** Which of the following represents the inverse of the linear function  $y = \frac{2}{3}x + 8$ ?

(1)  $y = \frac{3}{2}x - 8$

(3)  $y = -\frac{3}{2}x + 8$

(2)  $y = \frac{3}{2}x - 12$

(4)  $y = -\frac{3}{2}x + 12$

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**Exercise #4:** What is the  $y$ -intercept of the inverse of  $y = \frac{3}{5}x - 9$ ?

(1)  $y = 15$

(3)  $y = 9$

(2)  $y = \frac{1}{9}$

(4)  $y = -\frac{5}{3}$

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Sometimes we are asked to work with linear functions in their point-slope form. The method of finding the inverse and plotting it, though, do not change just because the linear equation is written in a different form.

**Exercise #5:** Which of the following would be an equation for the inverse of  $y + 6 = 4(x - 2)$ ?

(1)  $y - 2 = \frac{1}{4}(x + 6)$

(3)  $y - 6 = -4(x + 2)$

(2)  $y - 2 = -\frac{1}{4}(x + 6)$

(4)  $y + 2 = -4(x - 6)$

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**Exercise #6:** Which of the following points lies on the graph of the inverse of  $y - 8 = 5(x + 2)$ ?

(1)  $(8, -2)$

(3)  $(-10, 40)$

(2)  $(-8, 2)$

(4)  $(-2, 8)$

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**Exercise #7:** Which of the following linear functions would *not* have an inverse that is also a function?

(1)  $y = x$

(3)  $y = 2$

(2)  $2y = x$

(4)  $y = 5x - 1$

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**INVERSES OF LINEAR FUNCTIONS**  
**ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK**

**SKILLS**

1. The graph of a function and its inverse are always symmetric across which of the following lines?

(1)  $y = 0$

(3)  $y = x$

(2)  $x = 0$

(4)  $y = 1$

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2. Which of the following represents the inverse of the linear function  $y = 3x - 24$ ?

(1)  $y = \frac{1}{3}x + 8$

(3)  $y = -\frac{1}{3}x + 24$

(2)  $y = -\frac{1}{3}x - 8$

(4)  $y = \frac{1}{3}x - \frac{1}{24}$

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3. If the  $y$ -intercept of a linear function is 8, then we know which of the following about its inverse?

(1) Its  $y$ -intercept is  $-8$ .

(3) Its  $y$ -intercept is  $\frac{1}{8}$ .

(2) Its  $x$ -intercept is 8.

(4) Its  $x$ -intercept is  $-8$ .

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4. If both were plotted, which of the following linear functions would be parallel to its inverse?

(1)  $y = 2x$

(3)  $y = 5x - 1$

(2)  $y = \frac{2}{3}x - 4$

(4)  $y = x + 6$

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5. Which of the following represents the equation of the inverse of  $y = \frac{4}{3}x + 24$ ?

(1)  $y = -\frac{4}{3}x - 24$

(3)  $y = \frac{3}{4}x - 18$

(2)  $y = -\frac{3}{4}x + 18$

(4)  $y = \frac{4}{3}x - 24$

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6. Which of the following points lies on the inverse of  $y + 2 = 4(x - 1)$ ?

(1)  $(2, -1)$

(3)  $\left(\frac{1}{2}, 1\right)$

(2)  $(-1, 2)$

(4)  $(-2, 1)$

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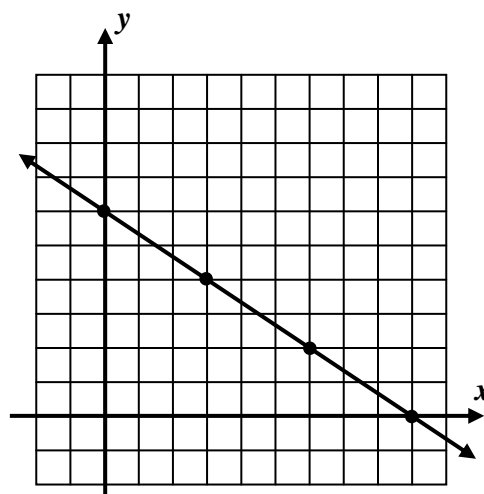
7. A linear function is graphed below. Answer the following questions based on this graph.

(a) Write the equation of this linear function in  $y = mx + b$  form.

(b) Sketch a graph of the inverse of this function on the same grid.

(c) Write the equation of the inverse in  $y = mx + b$  form.

(d) What is the intersection point of this line with its inverse?



## APPLICATIONS

8. A car traveling at a constant speed of 58 miles per hour has a distance of  $y$ -miles from Poughkeepsie, NY, given by the equation  $y = 58x + 24$ , where  $x$  represents the time in hours that the car has been traveling.

(a) Find the equation of the inverse of this linear function in  $y = \frac{x-a}{b}$  form.

(b) Evaluate the function you found in part (a) for an input of  $x = 227$ .

(c) Give a physical interpretation of the answer you found in part (b). Consider what the input and output of the inverse represent in order to answer this question.

## REASONING

9. Given the general linear function  $y = mx + b$ , find an equation for its inverse in terms of  $m$  and  $b$ .

