

## SUMMATION NOTATION

### ALGEBRA 2 WITH TRIGONOMETRY

Much of our work in this unit and the next will deal with situations where a large quantity of numbers will need to be added. In order to specify this addition or summarize it, we introduce a new notation, known as **summation or sigma notation** that will represent these sums.

#### SUMMATION (SIGMA) NOTATION

$$\sum_{i=a}^n f(i) = f(a) + f(a+1) + f(a+2) + \cdots + f(n)$$

where  $i$  is called the **index variable**, which starts at a value of  $a$ , ends at a value of  $n$ , and moves by unit increments.

**Exercise #1:** Evaluate each of the following sums.

(a)  $\sum_{i=3}^5 2i$

(b)  $\sum_{k=-1}^3 k^2$

(c)  $\sum_{j=-2}^2 2^j$

(d)  $\sum_{i=1}^5 (-1)^i$

(e)  $\sum_{k=0}^4 (2k+1)$

(f)  $\sum_{i=1}^3 i(i+1)$

**Exercise #2:** Which of represents the value of  $\sum_{i=1}^4 \frac{1}{i}$ ?

(1)  $\frac{1}{10}$

(3)  $\frac{25}{12}$

(2)  $\frac{9}{4}$

(4)  $\frac{31}{24}$



**Exercise #3:** Consider the sum given by  $\sum_{i=0}^3 {}_3C_i x^{3-i} y^i$ .

(a) Write out this summation in simplest form.

(b) Write a binomial expression that this sum represents.

It is also good to be able to place sums into sigma notation. These answers, though, will not be unique.

**Exercise #4:** Express each sum using sigma notation. Use  $i$  as your index variable.

(a)  $9 + 16 + 25 + \dots + 100$

(b)  $-6 + -3 + 0 + 3 + \dots + 15$

(c)  $\frac{1}{25} + \frac{1}{5} + 1 + 5 + \dots + 625$

**Exercise #5:** Which of the following represents the sum  $3 + 6 + 12 + 24 + 48$ ?

(1)  $\sum_{i=1}^5 3^i$

(3)  $\sum_{i=0}^4 6^{i-1}$

(2)  $\sum_{i=0}^4 3(2)^i$

(4)  $\sum_{i=3}^{48} i$

**Exercise #6:** Some sums are more interesting than others. Determine the value of  $\sum_{i=1}^{99} \left( \frac{1}{i} - \frac{1}{i+1} \right)$ . Show your reasoning. This is known as a **telescoping series (or sum)**.



**SUMMATION NOTATION**  
**ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK**

**SKILLS**

1. Evaluate each of the following. Place any non-integer answer in simplest rational form.

(a)  $\sum_{i=2}^5 4i$

(b)  $\sum_{k=0}^3 (k^2 + 1)$

(c)  $\sum_{j=-2}^0 (2j + 1)$

(d)  $\sum_{i=-1}^3 2^i$

(e)  $\sum_{k=0}^4 (-1)^{2k+1}$

(f)  $\sum_{i=1}^3 \log(10^i)$

(g)  $\sum_{n=1}^4 \frac{n}{n+1}$

(h)  $\sum_{r=0}^4 {}_4C_r$

(i)  $\sum_{k=0}^3 256^{\frac{1}{2^k}}$

2. Which of the following is the value of  $\sum_{k=0}^4 (4k + 1)$ ?

(1) 53

(3) 37

(2) 45

(4) 80

3. The sum  $\sum_{i=4}^7 2^{i-7}$  is equal to

(1)  $\frac{15}{8}$

(3)  $\frac{3}{4}$

(2)  $\frac{3}{2}$

(4)  $\frac{7}{8}$



4. Write each of the following sums using sigma notation. Use  $k$  as your index variable. Note, there are many correct ways to write each sum (and even more incorrect ways).

(a)  $-2 + 4 + -8 + \dots + 64 + -128$

(b)  $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{81} + \frac{1}{100}$

(c)  $4 + 9 + 14 + \dots + 44 + 49$

5. Which of the following represents the sum  $2 + 5 + 10 + \dots + 82 + 101$ ?

(1)  $\sum_{j=1}^6 (4j - 3)$

(3)  $\sum_{j=1}^{10} (j^2 + 1)$

(2)  $\sum_{j=3}^{103} (j - 2)$

(4)  $\sum_{j=0}^{11} (4^j + 1)$

### REASONING

6. A curious pattern occurs when we look at the behavior of the sum  $\sum_{k=1}^n (2k - 1)$ .

(a) Find the value of this sum for a variety of values of  $n$  below:

$$n = 2: \sum_{k=1}^2 (2k - 1) =$$

$$n = 4: \sum_{k=1}^4 (2k - 1) =$$

$$n = 3: \sum_{k=1}^3 (2k - 1) =$$

$$n = 5: \sum_{k=1}^5 (2k - 1) =$$

(b) What types of numbers are you summing?  
What types of numbers are the sums?

(c) Find the value of  $n$  such that  $\sum_{k=1}^n (2k - 1) = 196$ .

