Name: ___________________________________ Date: _________________

BINOMIAL EXPANSIONS
ALGEBRA 2 WITH TRIGONOMETRY

Our final topic in this chapter on probability has little to do with probability, but has parallels to binomial probability. In this lesson, we learn to how quickly expand (write out) expressions of the form $(x+y)^n$, where n is a non-negative integer. This is called **expanding a binomial** and is an important skill needed for calculus. In the first exercise, we simply develop some patterns.

***Exercise* #1:** Consider expressions of the form $(x+y)^n$, for values of n that are non-negative integers.

(a) Quickly expand each of the following:

$(x+y)^0 =$

$(x+y)^1 =$

$(x+y)^2 =$

(b) Using your answer to $(x+y)^2$, expand $(x+y)^3$

(c) What is true about the powers of x? What is true about the powers of y?

(d) What is true about the sum of the powers of any given term?

Based on the observations from the *Exercise* #1, it is simple to predict the power pattern of any binomial expansion. The only question becomes the determination of the coefficients on the individual terms. Interestingly enough, based on a counting argument, these coefficients are simply combinations. The overall pattern is given below:

BINOMIAL EXPANSION PATTERN

$$(x+y)^n = {}_nC_0\, x^n + {}_nC_1\, x^{n-1}y + {}_nC_2\, x^{n-2}y^2 + \cdots + {}_nC_{n-1}\, xy^{n-1} + {}_nC_n\, y^n$$

***Exercise* #2:** Use the pattern above to expand $(x+y)^4$ and $(x+y)^5$.

(a) $(x+y)^4 =$

(b) $(x+y)^5 =$

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Binomial expansion problems can become more challenging when they involve more complicated binomial expressions.

***Exercise* #3:** Completely expand $(2x-1)^4$. Write it in simplest standard form. Check using a table on your calculator.

Many times only a single term of an expansion is needed. Typically, the whole pattern does not need to be written out to determine this one term.

***Exercise* #4:** Which of the following represents the third term, when written in standard form, of $(3x+2y)^6$?

(1) $4320x^3y^3$ (3) $90x^4y^2$

(2) $55x^3y^3$ (4) $4860x^4y^2$

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***Exercise* #5:** When written in standard form, which of the following represents the middle term of the expansion of the expression $(2x-3)^8$?

(1) $-52560x^4$ (3) $90720x^4$

(2) $-90720x^4$ (4) $52560x^4$

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***Exercise* #6:** If written in standard form, the last term of the expansion of $(3x-2y)^6$ is

(1) $64y^6$ (3) $-64y^6$

(2) $2y^6$ (4) $-2y^6$

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***Exercise* #7:** Which of the following represents the exponent of x on the second term of the expansion of $\left(x^2-\frac{1}{x}\right)^4$ when written in simplest standard form?

(1) 7 (3) 3

(2) 5 (4) 4

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BINOMIAL EXPANSIONS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Expand each of the following expressions. Express your answers in simplest standard form.

(a) $(x+2)^4$

(b) $(2x-3y)^3$

2. Which of the following represents the 4th term of the expansion of $(4x+3)^6$?

(1) $34560x^4$ (3) $34560x^3$

(2) $111x^4$ (4) $111x^3$

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3. If written in standard form, the middle term of the expansion of $(3x-4y)^4$ is

(1) $31x^2y^2$ (3) $-31x^2y^2$

(2) $-864x^2y^2$ (4) $864x^2y^2$

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4. If simplified and written in standard form, the fourth term of $\left(x^2+\frac{1}{x^2}\right)^{10}$ is

(1) $210x^6$ (3) $120x^8$

(2) $120x^4$ (4) $210x^{10}$

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APPLICATIONS

5. In calculus, one of the foundational expressions is known as the **difference quotient**. For any function, the difference quotient is defined as:

$$\frac{f(x+h)-f(x)}{h}$$

For example, if $f(x)=x^2$, then we would have:

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-x^2}{h}=\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}=\frac{2xh+h^2}{h}=\frac{\cancel{h}(2x+h)}{\cancel{h}}=2x+h \;\; (\text{as long as } h\neq 0)$$

(a) Evaluate the difference quotient for $f(x)=x^3$

(b) Evaluate the difference quotient for $f(x)=x^4$

REASONING

6. Consider the expression $(8)^4-4(8)^3(7)+6(8)^2(7)^2-4(8)(7)^3+(7)^4$.

(a) Find the value of this expression by using your calculator.

(b) Explain why this value makes sense by rewriting this expression as an unexpanded binomial expression.

7. In binomial probability, p represents the probability of success and q represents the probability of failure. Consider a binomial probability experiment with three trials.

(a) Expand $(p+q)^3$.

(b) What does the sum of your expansion in (a) equal? What does it represent?